

Deriving the equations of motion

ASHPHYS · REVISION CHEAT SHEET

AS Level

9702 AS Ch.2

Chapter 2: Accelerated motion

Lesson 2.9

Sections 2.9

Goal: Derive the four suvat equations from a velocity–time graph.

01 The big picture

For uniform acceleration the v - t graph is a straight line from u to v : its gradient is a and the area under it is s .

Those give the first two equations; algebra gives the other two.

Uniformly accelerated motion [paraphrase]

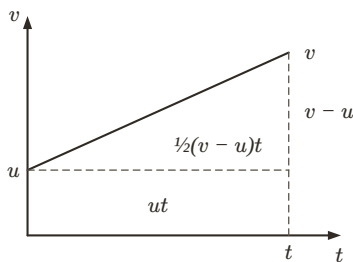
Constant acceleration in a straight line.

02 Deriving the equations

D1 From the v - t graph

Velocity rises uniformly from u to v in time t .

Fig. 1 · The area under a uniform v - t line



Rectangle ut plus triangle $\frac{1}{2}(v-u)t$; the gradient is a .

1. Gradient = acceleration: $a = \frac{v-u}{t} \Rightarrow v = u + at$

2. Area = rectangle + triangle: $s = ut + \frac{1}{2}(v-u)t = \frac{1}{2}(u+v)t$

3. Put $v-u = at$ into the area: $s = ut + \frac{1}{2}at^2$

4. Put $t = (v-u)/a$ into the area: $s = \frac{(v+u)(v-u)}{2a} \Rightarrow v^2 = u^2 + 2as$

RESULT

$$v = u + at, \quad s = \frac{1}{2}(u+v)t,$$

$$s = ut + \frac{1}{2}at^2, \quad v^2 = u^2 + 2as$$

03 Sample problems with solutions

Sample 1 · Area check

An object accelerates uniformly from 4.0 m s^{-1} to 10 m s^{-1} in 3.0 s . Find s from the area, then check it with $s = ut + \frac{1}{2}at^2$.

Step 1. Area (trapezium):

$$s = \frac{1}{2}(4.0 + 10)(3.0) = 21 \text{ m}$$

Step 2. Check: $a = (10 - 4.0)/3.0 = 2.0 \text{ m s}^{-2}$,
so

$$s = 4.0(3.0) + \frac{1}{2}(2.0)(3.0)^2 = 12 + 9.0 = 21 \text{ m} \checkmark$$

Answer: $s = 21 \text{ m}$ both ways

04 Practice problems

P1 Which feature of a v - t graph gives $v = u + at$?

Answer: The gradient

P2 Which suvat equation has no t , and how is it derived?

Answer: $v^2 = u^2 + 2as$ (eliminate t)

P3 In $s = ut + \frac{1}{2}at^2$, which part of the graph area is ut ?

Answer: The rectangle under u

P4 Find s from the area: $u = 0$, $v = 12 \text{ m s}^{-1}$, $t = 5.0 \text{ s}$.

Answer: 30 m

05 Other useful information

Common mistakes

- ✗ **Wrong:** Use suvat for any motion.
- ✓ **Right:** Only for constant acceleration.

Exam tips

- Forgot an equation? Sketch the v - t trapezium.

What the syllabus asks

- 9702 topic 2: derive the suvat equations. Coursebook 2.9.

Explore: [Practice questions for this lesson](#)