

Understanding projectiles

Goal: Solve problems for a projectile launched at an angle by resolving its initial velocity.

velocity

01 The big picture

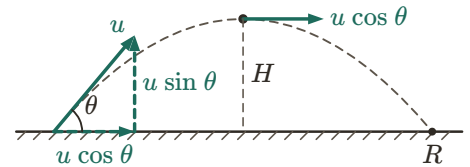
Resolve the launch velocity into $u \cos \theta$ horizontally (constant all flight) and $u \sin \theta$ vertically (decelerated by g on the way up). At the highest point the vertical velocity is zero, but the horizontal velocity is not.

Over level ground the flight is symmetrical: time up = time down. Without air resistance the range is greatest at 45° . Air resistance lowers the height and the range and makes the descent steeper.

Range [paraphrase]

The horizontal distance travelled before landing.

Fig. 1 · Launched at an angle



At the top only $u \cos \theta$ is left: the vertical velocity is zero.

02 Key equations

Components

$$u_x = u \cos \theta, \quad u_y = u \sin \theta$$

Level ground

$$T = \frac{2u_y}{g}, \quad R = u_x T$$

Max height

$$H = \frac{u_y^2}{2g}$$

Symbols and units

u, θ launch speed and angle $\text{m s}^{-1}, ^\circ$ T time of flight s H, R max height, range m

03 Sample problems with solutions

Sample 1 · A kicked ball

A ball is kicked at 20 m s^{-1} at 30° above level ground. Find its time of flight, range and maximum height.

Step 1. Components:

$$u_x = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}, \quad u_y = 20 \sin 30^\circ = 10.0 \text{ m s}^{-1}$$

Step 2. Vertical gives the time; horizontal the range:

$$T = \frac{2 \times 10.0}{9.81} = 2.04 \text{ s}, \quad R = 17.3 \times 2.04 = 35.3 \text{ m}$$

Step 3. At the top $v_y = 0$:

$$H = \frac{10.0^2}{2 \times 9.81} = 5.10 \text{ m}$$

Answer: 2.0 s; 35 m; 5.1 m

Unit check: $\frac{(\text{m s}^{-1})^2}{\text{m s}^{-2}} = \text{m} \checkmark$

04 Practice problems

- P1** What is the velocity of a projectile at its highest point?
Answer: $u \cos \theta$, horizontal
- P2** Thrown at 25 m s^{-1} at 60° : find the maximum height.
Answer: 23.9 m (24 m)
- P3** At 15 m s^{-1} and 40° : find the time of flight.
Answer: 1.97 s (2.0 s)
- P4** Which launch angle gives the longest range?
Answer: 45° (no air resistance)

05 Other useful information

Common mistakes

- ✗ **Wrong:** Use u in the vertical equations.
- ✓ **Right:** Vertical: $u \sin \theta$. Horizontal: $u \cos \theta$.

Exam tips

- Find T from the vertical motion first; it links the two directions.

What the syllabus asks

- 9702 topic 2: projectile motion without air resistance. Coursebook 2.14.

Explore: Practice questions for this lesson