

Combining velocities

AS Level

9702 AS Ch.1

Chapter 1: Kinematics

Lesson 1.6

Sections 1.6

Goal: Add velocities as vectors, for boats crossing rivers and aircraft in a wind.

velocity

01 The big picture

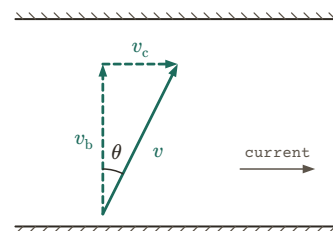
When something moves through a medium that is itself moving (a boat in a current, a plane in a wind), its velocity relative to the ground is the vector sum of the two velocities.

The parts act independently: the time to cross a river depends only on the velocity across it, and the current only carries the boat downstream while it crosses.

Resultant velocity [paraphrase]

The vector sum of the velocities: the actual velocity relative to the ground.

Fig. 1 · A boat crossing a river



The boat heads straight across (v_b); the current (v_c) carries it downstream, so it actually moves along v .

02 Key equations

Resultant (perpendicular)

$$v = \sqrt{v_b^2 + v_c^2}$$

Angle from straight across

$$\tan \theta = \frac{v_c}{v_b}$$

Time to cross

$$t = \frac{w}{v_b}$$

Symbols and units

v_b boat velocity through the water m s^{-1} v_c velocity of the current m s^{-1} w width of the river m

03 Sample problems with solutions

Sample 1 · Crossing a river

A boat heads straight across a river 60 m wide at 3.0 m s^{-1} through the water. The current flows at 1.5 m s^{-1} . Calculate the boat's resultant velocity, the time to cross and how far downstream it lands.

Step 1. Resultant velocity:

$$v = \sqrt{3.0^2 + 1.5^2} = 3.35 \text{ m s}^{-1}, \quad \theta = \tan^{-1} \frac{1.5}{3.0} = 27^\circ$$

Step 2. Only the across-velocity crosses the river; meanwhile the current carries it downstream:

$$t = \frac{60}{3.0} = 20 \text{ s}, \quad x = 1.5 \times 20 = 30 \text{ m}$$

Answer: 3.4 m s^{-1} at 27° downstream of straight across; 20 s; 30 m downstream

Unit check: $\frac{\text{m}}{\text{m s}^{-1}} = \text{s} \checkmark$

04 Practice problems

P1 A plane flies north at 200 m s^{-1} in a wind blowing east at 50 m s^{-1} . Find its resultant velocity.

Answer: 206 m s^{-1} at 14° east of north

P2 To land directly opposite, at what angle upstream must the sample's boat head? What is its speed across?

Answer: 30° upstream; 2.6 m s^{-1}

P3 A passenger walks forwards at 1.5 m s^{-1} in a train moving at 20 m s^{-1} . Find their speed over the ground.

Answer: 21.5 m s^{-1}

P4 A swimmer crosses a 48 m river at 1.2 m s^{-1} across a 0.50 m s^{-1} current. Find the crossing time.

Answer: 40 s

05 Other useful information

Common mistakes

- ✗ **Wrong:** Use the resultant speed to find the crossing time.
- ✓ **Right:** Use only the velocity component across the river.

Exam tips

- Perpendicular velocities add by Pythagoras, not arithmetic. Say which angle you give.

What the syllabus asks

- 9702 topic 1: add coplanar vectors (velocities). Coursebook 1.6.

Explore: [Practice questions for this lesson](#)