

Combining displacements

AS Level

9702 AS Ch.1

Chapter 1: Kinematics

Lesson 1.5

Sections 1.5

Goal: Add two vectors tip to tail, and split one vector into two perpendicular components.

 velocity

01 The big picture

To add vectors, draw them tip to tail: the resultant runs from the start of the first to the end of the last. For two perpendicular vectors use Pythagoras and $\tan \theta$; for any other angle use a scale drawing.

Resolving does the reverse: one vector at angle θ to the x direction is replaced by two perpendicular components that together have the same effect.

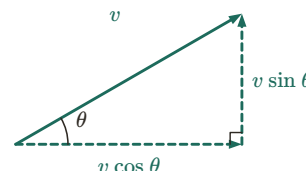
Resultant [paraphrase]

The single vector with the same effect as two or more vectors together.

Components [paraphrase]

Two perpendicular vectors that add up to the original vector.

Fig. 1 · Resolving into components



$v \cos \theta$ is adjacent to the angle; $v \sin \theta$ is opposite it.

02 Key equations

Resultant of perpendicular vectors

$$R = \sqrt{a^2 + b^2}$$

Its direction

$$\tan \theta = \frac{b}{a}$$

Components

$$v_x = v \cos \theta, \quad v_y = v \sin \theta$$

Symbols and units

a, b two perpendicular vectors (their unit) R resultant (same unit) θ angle to the x direction $^\circ$

03 Sample problems with solutions

Sample 1 · Resolving a velocity

A ball is kicked at 20 m s^{-1} at 30° above the horizontal. Calculate the horizontal and vertical components of its velocity.

Step 1. Horizontal (adjacent to θ):

$$v_x = 20 \cos 30^\circ = 17.3 \text{ m s}^{-1}$$

Step 2. Vertical (opposite θ):

$$v_y = 20 \sin 30^\circ = 10.0 \text{ m s}^{-1}$$

Answer: 17 m s^{-1} horizontally, 10 m s^{-1} vertically

Unit check: components keep the vector's unit: m s^{-1} ✓

04 Practice problems

P1 Find the resultant of 12 N east and 5.0 N north.

Answer: 13 N at 23° north of east

P2 Resolve a 50 N force at 40° above the horizontal.

Answer: 38 N horizontal, 32 N vertical

P3 Two forces of 3 N and 4 N act on a point. What are the largest and smallest possible resultants?

Answer: 7 N and 1 N

P4 Find the vertical component of a velocity of 8.0 m s^{-1} at 60° to the horizontal.

Answer: 6.9 m s^{-1}

05 Other useful information

Common mistakes

✗ **Wrong:** Add the sizes of two vectors at an angle ($3 \text{ N} + 4 \text{ N} = 7 \text{ N}$).

✓ **Right:** Only when they point the same way. At 90° : $\sqrt{(3^2 + 4^2)} = 5 \text{ N}$.

✗ **Wrong:** Use sin for the side next to the angle.

✓ **Right:** Adjacent side: $\cos \theta$. Opposite side: $\sin \theta$.

Exam tips

- Always draw the vector triangle and mark θ before choosing sin or cos.

What the syllabus asks

- 9702 topic 1: add and subtract coplanar vectors; represent a vector as two perpendicular components. Coursebook 1.5.

Explore: [Vector Addition Sandbox \(simulation\)](#) · [Practice questions for this lesson](#)