

Goal: Use $F = GMm/r^2$, $g = GM/r^2$ and $\phi = -GM/r$, and analyse circular orbits, including geostationary satellites.

— force — velocity

01 The big picture

A gravitational field is a region in which a mass feels a force. Its strength at a point is the force per unit mass on a small test mass placed there, $g = F/m$, in N kg^{-1} . The force is always attractive.

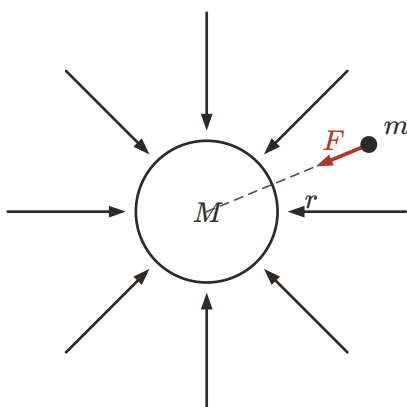
Two point masses attract with $F = GMm/r^2$: proportional to each mass and inversely proportional to the square of the distance between their centres. Outside a uniform sphere, the sphere acts as if all its mass were at its centre, so r is always measured from the centre. Dividing by m gives the field of a point mass, $g = GM/r^2$.

Field lines show the direction of the force on a mass; their spacing shows the strength. Round a planet they are radial and point inwards. Near the surface they are almost parallel and equally spaced: the field is nearly uniform, so $g \approx 9.81 \text{ N kg}^{-1}$ and $\Delta E_p = mg\Delta h$ work only for heights much smaller than the radius.

Gravitational potential $\phi = -GM/r$ is zero at infinity and negative everywhere else: work must be done to pull a mass away. The potential energy of a mass m is $E_p = m\phi = -GMm/r$, and moving it changes its energy by $m\Delta\phi$.

For an orbit, gravity provides the centripetal force. So $GMm/r^2 = mv^2/r$, giving $v = \sqrt{GM/r}$ and $T^2 \propto r^3$, whatever the satellite's mass.

Fig. 1 · Radial field of a planet



Field lines point inwards, towards the centre. They spread out with distance: the field gets weaker, as $1/r^2$.

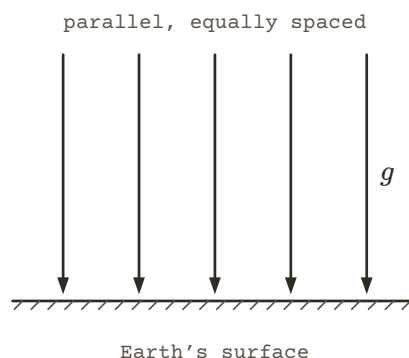
Gravitational field strength [paraphrase]

The gravitational force per unit mass on a small test mass at that point, $g = F/m$.

Newton's law of gravitation [paraphrase]

Two point masses attract each other with a force proportional to the product of their masses and inversely proportional to the square of their separation.

Fig. 2 · Near the surface: uniform



Over heights much smaller than the radius, the field lines are parallel and equally spaced, so g is constant.

Gravitational potential (syllabus) [syllabus wording]

"The work done per unit mass in bringing a small test mass from infinity to the point."

Geostationary orbit (syllabus) [syllabus wording]

"A satellite in a geostationary orbit remains at the same point above the Earth's surface, with an orbital period of 24 hours, orbiting from west to east, directly above the Equator."

02 Deriving the equations

D1 Field strength of a point mass

Put a small test mass m at distance r from the centre of a mass M , and use the definition $g = F/m$.

1. Newton's law of gravitation gives the force on the test mass:

$$F = \frac{GMm}{r^2}$$

2. Field strength is force per unit mass, so divide by m ; the test mass cancels:

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

3. At the surface, $r = R$ and $g_0 = GM/R^2$. Dividing the two gives how g falls with distance (inverse square):

$$\frac{g}{g_0} = \frac{R^2}{r^2}$$

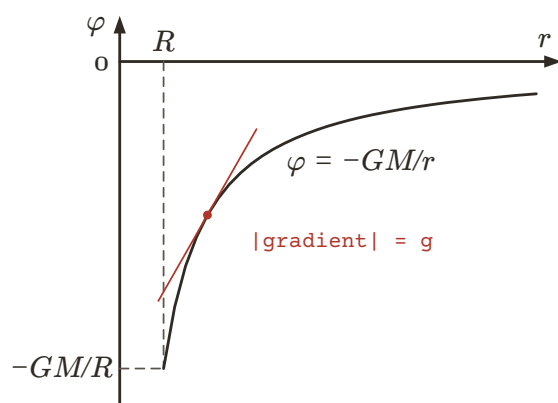
RESULT

$$g = \frac{GM}{r^2}$$

D2 Potential and potential energy

Potential is zero at infinity. A mass coming in from infinity is pulled by the field, so the work done on it per unit mass is negative.

Fig. 3 · Potential against distance



ϕ rises from $-GM/R$ at the surface towards zero at infinity. The gradient of the graph at any point is the field strength there.

1. The potential of a point mass:

$$\phi = -\frac{GM}{r}$$

2. The potential energy of a mass m at distance r is m times the potential:

$$E_p = m\phi = -\frac{GMm}{r}$$

3. Moving from r_1 out to r_2 raises the potential (towards zero), so it takes energy:

$$\Delta E_p = m\Delta\phi = GMm \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

4. The field strength is the size of the gradient of the potential-distance graph:

$$g = -\frac{\Delta\phi}{\Delta r}$$

RESULT

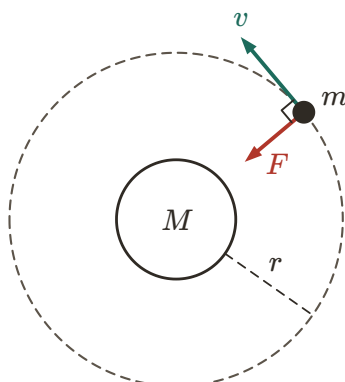
$$\phi = -\frac{GM}{r} \quad E_p = -\frac{GMm}{r}$$

02 Deriving the equations (continued)

D3 Circular orbits and $T^2 \propto r^3$

A satellite of mass m orbits a planet of mass M at radius r . Gravity is the only force on it, so gravity provides the centripetal force.

Fig. 4 · A satellite in a circular orbit



Gravity is the only force on the satellite: it points to the centre of the orbit and provides the centripetal force.

1. Gravitational force = centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

2. Cancel m and one r : the orbital speed does not depend on the satellite's mass.

$$v = \sqrt{\frac{GM}{r}}$$

3. Substitute $v = 2\pi r/T$ and square:

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r} \implies T^2 = \frac{4\pi^2}{GM} r^3$$

4. For satellites of the same planet $4\pi^2/GM$ is constant, so $T^2 \propto r^3$.

RESULT

$$v = \sqrt{\frac{GM}{r}} \quad T^2 = \frac{4\pi^2 r^3}{GM}$$

D4 Geostationary orbit

A geostationary satellite stays above the same point on the Equator: it orbits west to east with the Earth's rotation, with period $T = 24 \text{ h} = 86\,400 \text{ s}$.

1. Cross-multiply $T^2 = 4\pi^2 r^3 / (GM)$ and divide by $4\pi^2$:

$$r^3 = \frac{GMT^2}{4\pi^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 86\,400^2}{4\pi^2} = 7.53 \times 10^{22} \text{ m}^3$$

2. Take the cube root, then subtract the Earth's radius for the height:

$$r = 4.22 \times 10^7 \text{ m}, \quad h = r - R = 3.59 \times 10^7 \text{ m}$$

RESULT

$$r \approx 4.2 \times 10^7 \text{ m} \quad (h \approx 3.6 \times 10^7 \text{ m})$$

03 Key equations

Field strength (definition)

$$g = \frac{F}{m}$$

Newton's law of gravitation

$$F = \frac{GMm}{r^2}$$

Field of a point mass

$$g = \frac{GM}{r^2}$$

r from the centre

Gravitational potential

$$\varphi = -\frac{GM}{r}$$

zero at infinity

Potential energy

$$E_p = m\varphi = -\frac{GMm}{r}$$

Change in potential energy

$$\Delta E_p = m \Delta \varphi$$

Orbital speed

$$v = \sqrt{\frac{GM}{r}}$$

from $GMm/r^2 = mv^2/r$

Orbital period ($T^2 \propto r^3$)

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Near the surface only

$$\Delta E_p = mg \Delta h$$

when $\Delta h \ll R$

Symbols and units

G	gravitational constant, 6.67×10^{-11}	$\text{N m}^2 \text{kg}^{-2}$	M	mass of the planet or star	kg	m	mass of the test mass or satellite	kg
r	distance from the centre	m	R	radius of the planet	m	g	gravitational field strength	N kg^{-1}
φ	gravitational potential	J kg^{-1}	E_p	gravitational potential energy	J	v	orbital speed	m s^{-1}
T	orbital period	s	M_E	Earth's mass, 5.97×10^{24}	kg	R_E	Earth's radius, 6.37×10^6	m

04 Sample problems with solutions

Sample 1 · Field strength above the surface

The International Space Station orbits 4.1×10^5 m above the Earth's surface. Calculate the gravitational field strength there. Use the Earth data in the symbols list.

Step 1. r is measured from the centre of the Earth, not from the surface:

$$r = R + h = 6.37 \times 10^6 + 4.1 \times 10^5 = 6.78 \times 10^6 \text{ m}$$

Step 2. Substitute into $g = GM/r^2$:

$$g = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.78 \times 10^6)^2} = 8.66 \text{ N kg}^{-1}$$

Step 3. Sense check: a little less than 9.81 N kg^{-1} . Astronauts feel weightless because they are in free fall, not because g is zero.

Answer: $g = 8.7 \text{ N kg}^{-1}$

Unit check: $\frac{\text{N m}^2 \text{kg}^{-2} \times \text{kg}}{\text{m}^2} = \text{N kg}^{-1} \checkmark$

04 Sample problems with solutions (continued)

Sample 2 · Energy to lift a probe a long way up

A 500 kg probe is lifted from the Earth's surface to a height equal to the Earth's radius. Calculate the increase in its gravitational potential energy.

Step 1. Potential at the surface ($r = R$) and at height R ($r = 2R$):

$$\varphi_R = -\frac{GM}{R} = -6.25 \times 10^7 \text{ J kg}^{-1}, \quad \varphi_{2R} = -3.13 \times 10^7 \text{ J kg}^{-1}$$

Step 2. Change in potential energy:

$$\Delta E_p = m\Delta\varphi = 500 \times 3.13 \times 10^7 = 1.56 \times 10^{10} \text{ J}$$

Step 3. Why not mgh ? It gives $3.1 \times 10^{10} \text{ J}$, twice too much, because g falls to a quarter of its surface value over this height.

Answer: $\Delta E_p = 1.6 \times 10^{10} \text{ J}$

Unit check: $\text{kg} \times \text{J kg}^{-1} = \text{J} \checkmark$

Sample 3 · Speed and period of a satellite

A satellite orbits $2.0 \times 10^6 \text{ m}$ above the Earth's surface. Calculate its orbital speed and its period.

Step 1. Radius of the orbit, from the centre:

$$r = 6.37 \times 10^6 + 2.0 \times 10^6 = 8.37 \times 10^6 \text{ m}$$

Step 2. Gravity provides the centripetal force, so:

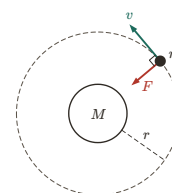
$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{8.37 \times 10^6}} = 6.90 \times 10^3 \text{ m s}^{-1}$$

Step 3. One orbit is a distance $2\pi r$:

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 8.37 \times 10^6}{6.90 \times 10^3} = 7.62 \times 10^3 \text{ s}$$

Answer: $v = 6.9 \times 10^3 \text{ m s}^{-1}$, $T = 7.6 \times 10^3 \text{ s}$ (about 2.1 hours)

Unit check: $\sqrt{\frac{\text{N m}^2 \text{ kg}^{-2} \times \text{kg}}{\text{m}}} = \sqrt{\text{m}^2 \text{ s}^{-2}} = \text{m s}^{-1} \checkmark$



see Fig. 4

Sample 4 · Weighing the Earth with the Moon

The Moon orbits the Earth at a radius of $3.84 \times 10^8 \text{ m}$ with a period of 27.3 days. Calculate the mass of the Earth.

Step 1. Period in seconds:

$$T = 27.3 \times 86\,400 = 2.36 \times 10^6 \text{ s}$$

Step 2. M is in the denominator and T^2 is the lone term on the other side: swap them in one step (cross-multiplication).

$$T^2 = \frac{4\pi^2 r^3}{GM} \implies M = \frac{4\pi^2 r^3}{GT^2}$$

Step 3. Substitute:

$$M = \frac{4\pi^2 \times (3.84 \times 10^8)^3}{6.67 \times 10^{-11} \times (2.36 \times 10^6)^2} = 6.02 \times 10^{24} \text{ kg}$$

Answer: $M = 6.0 \times 10^{24} \text{ kg}$ (close to the accepted $5.97 \times 10^{24} \text{ kg}$)

Unit check: $\frac{\text{m}^3}{\text{N m}^2 \text{ kg}^{-2} \times \text{s}^2} = \frac{\text{m}^3 \text{ kg}^2}{\text{kg m}^3} = \text{kg} \checkmark$

05 Practice problems

- P1** Calculate the gravitational force between two 70 kg people whose centres are 1.0 m apart.
Answer: 3.3×10^{-7} N
- P3** A planet's field strength at distance r is g . What is it at distance $3r$?
Answer: $g/9$
- P5** Calculate the gravitational potential at the Earth's surface.
Answer: -6.3×10^7 J kg $^{-1}$
- P7** Calculate the speed of a satellite in a circular orbit of radius 7.0×10^6 m round the Earth.
Answer: 7.5×10^3 m s $^{-1}$
- P9** Phobos orbits Mars at a radius of 9.38×10^6 m with a period of 7.65 h. Calculate the mass of Mars.
Answer: 6.4×10^{23} kg
- P2** The Moon has mass 7.35×10^{22} kg and radius 1.74×10^6 m. Calculate the field strength at its surface.
Answer: 1.6 N kg $^{-1}$
- P4** At what height above the Earth's surface is g half its surface value?
Answer: 2.6×10^6 m
- P6** The potential at a point is -4.0×10^7 J kg $^{-1}$. Calculate the potential energy of a 200 kg satellite there.
Answer: -8.0×10^9 J
- P8** Two satellites orbit the same planet; one orbit has 4 times the radius of the other. Calculate the ratio of their periods.
Answer: 8
- P10** Calculate the height above the Earth's surface of a geostationary satellite.
Answer: 3.6×10^7 m

06 Other useful information

Common mistakes

- ✗ **Wrong:** Use the height above the surface as r .
- ✓ **Right:** r is the distance from the centre: $r = R + h$.
- ✗ **Wrong:** Use mgh for any change in height.
- ✓ **Right:** Only when g is constant ($h \ll R$). Otherwise use $\Delta E_p = m\Delta\phi$.
- ✗ **Wrong:** Astronauts float because there is no gravity in orbit.
- ✓ **Right:** g is still about 8.7 N kg $^{-1}$ at the ISS; they are in free fall with the station.

Exam tips

- Write $r = R + h$ at the start of every calculation above a planet.
- Orbits: start from $GMm/r^2 = mv^2/r$ (or $m\omega^2 r$), then cancel m .
- Ratio questions: use $g \propto 1/r^2$, $\phi \propto -1/r$ and $T^2 \propto r^3$; no constants needed.
- Convert periods to seconds: $24 \text{ h} = 86\,400 \text{ s}$.

Explore

- Practice: field strength of a point mass
- Practice: gravitational potential
- Practice: orbits (32 questions)

- ✗ **Wrong:** Gravitational potential is positive, like height.
- ✓ **Right:** It is zero at infinity and negative everywhere else; it rises towards zero as you move away.

- ✗ **Wrong:** A heavier satellite must orbit faster.
- ✓ **Right:** m cancels: $v = \sqrt{GM/r}$ depends only on M and r .

What the syllabus asks

- 9702 topic 13, Gravitational fields: field strength as force per unit mass, field lines, $F = Gm_1m_2/r^2$, $g = GM/r^2$, $\phi = -GM/r$, $E_p = -GMm/r$, circular orbits and geostationary orbits. Coursebook Chapter 17.
- A Level only: not part of AS (AS uses $\Delta E_p = mg\Delta h$ in a uniform field).
- Outside a uniform sphere, treat its mass as a point mass at its centre.
- Gravitational and electric fields are compared in Chapter 22.