

Coulomb's law and electric fields

9702 A Level Ch.22

Chapter 22: Coulomb's law

Lesson 22.1

Sections 22.1–22.5

A Level only (not AS)

Goal: Use $F = Qq/4\pi\epsilon_0 r^2$, $E = Q/4\pi\epsilon_0 r^2$ and $V = Q/4\pi\epsilon_0 r$ for point charges, combine fields and potentials, and compare with gravity.

— force

01 The big picture

Every charge sets up an electric field around it. The field strength at a point is the force per unit positive charge on a small test charge placed there, $E = F/q$, in N C^{-1} (the same as V m^{-1}). Like charges repel; unlike charges attract.

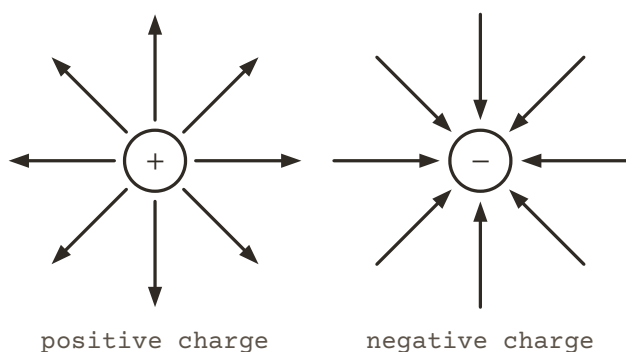
Two point charges exert a force $F = \frac{Qq}{4\pi\epsilon_0 r^2}$ on each other: Coulomb's law, another inverse-square law. Outside a charged conducting sphere, the sphere acts as if its charge were a point charge at its centre, so r is measured from the centre.

Dividing by q gives the field of a point charge, $E = \frac{Q}{4\pi\epsilon_0 r^2}$. It is radial: outwards from a positive charge, inwards towards a negative one. Fields are vectors, so fields from several charges add with their directions.

Electric potential $V = \frac{Q}{4\pi\epsilon_0 r}$ is zero at infinity, positive near a positive charge and negative near a negative one. It is a scalar, so potentials simply add with their signs. Moving a charge q through a potential difference ΔV takes energy $q\Delta V$, and the field is the negative potential gradient, $E = -\Delta V/\Delta r$.

The maths matches gravity ($F = GMm/r^2$) with one big difference: gravity only attracts, while electric forces can repel. Between two protons the electric force is about 10^{36} times the gravitational one.

Fig. 1 · Radial fields of point charges



Field lines point away from a positive charge and towards a negative one: the direction a positive test charge would be pushed.

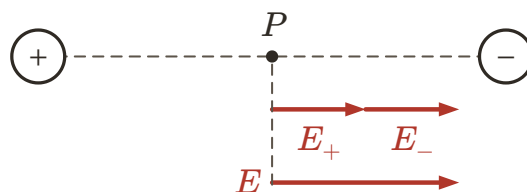
Electric field strength (syllabus) [syllabus wording]

"Force per unit positive charge."

Coulomb's law [paraphrase]

Two point charges exert a force on each other proportional to the product of the charges and inversely proportional to the square of their separation.

Fig. 2 · Fields add as vectors



Midway between $+Q$ and $-Q$ both fields point the same way (away from $+$, towards $-$), so they add; the potentials ($+$ and $-$) cancel.

Electric potential (syllabus) [syllabus wording]

"The work done per unit positive charge in bringing a small test charge from infinity to the point."

Field lines [paraphrase]

Show the direction of the force on a positive charge. They start on positive charges, end on negative charges, and are closer together where the field is stronger.

02 Deriving the equations

D1 Field strength of a point charge

Put a small positive test charge q at distance r from a point charge Q , and use the definition $E = F/q$.

1. Coulomb's law gives the force on the test charge:

$$F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

2. Field strength is force per unit positive charge, so divide by q ; the test charge cancels:

$$E = \frac{F}{q} = \frac{Q}{4\pi\epsilon_0 r^2}$$

3. Inverse square: doubling r quarters E . For a negative Q the field points inwards, towards the charge.

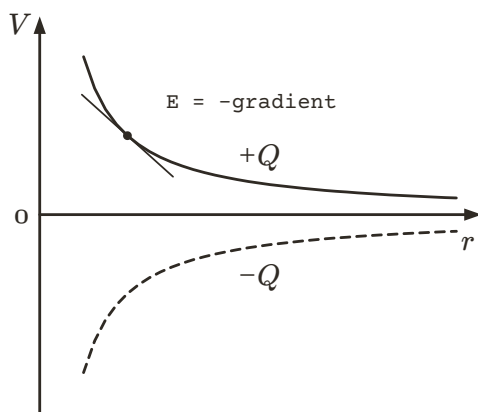
RESULT

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

D2 Potential, potential energy and the potential gradient

Potential is zero at infinity. Bringing a positive test charge towards a positive Q means pushing against the repulsion, so the work done, and the potential, are positive.

Fig. 3 · Potential against distance



Around $+Q$ the potential is positive and falls towards zero; around $-Q$ it is negative and rises towards zero. The field strength is minus the gradient.

1. The potential of a point charge (negative for a negative Q):

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

2. A charge q at that point has potential energy qV . The sign tells you the force: positive for repulsion, negative for attraction.

$$E_p = qV = \frac{Qq}{4\pi\epsilon_0 r}$$

3. Moving q through a potential difference ΔV :

$$\Delta E_p = q \Delta V$$

4. The field strength is the negative of the potential gradient: on a $V-r$ graph, E is minus the gradient.

$$E = -\frac{\Delta V}{\Delta r}$$

RESULT

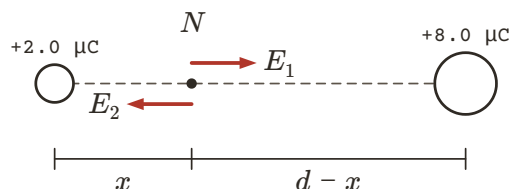
$$V = \frac{Q}{4\pi\epsilon_0 r} \quad E_p = \frac{Qq}{4\pi\epsilon_0 r}$$

02 Deriving the equations (continued)

D3 Combining two charges

With several charges, find each one's field and potential at the point, then combine them.

Fig. 4 · The neutral point



At N the field from each charge has the same size and points the opposite way, so the resultant field is zero.

1. Fields are vectors: draw each one (away from +, towards -) and add them with their directions.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

2. Potentials are scalars: add them with their signs.

$$V = \frac{Q_1}{4\pi\epsilon_0 r_1} + \frac{Q_2}{4\pi\epsilon_0 r_2}$$

3. Between two like charges there is a neutral point where the fields cancel. Distance x from Q_1 , with d between the charges:

$$\frac{Q_1}{x^2} = \frac{Q_2}{(d-x)^2} \Rightarrow \frac{d-x}{x} = \sqrt{\frac{Q_2}{Q_1}}$$

4. At the neutral point $E = 0$, but V is not zero: both potentials there are positive.

RESULT

add E as vectors, add V as scalars

D4 Electric and gravitational fields compared

Both are inverse-square fields, so every equation has a partner: replace GM by $Q/4\pi\epsilon_0$.

1. Force, field and potential side by side:

$$\frac{GMm}{r^2} \leftrightarrow \frac{Qq}{4\pi\epsilon_0 r^2}, \quad \frac{GM}{r^2} \leftrightarrow \frac{Q}{4\pi\epsilon_0 r^2}, \quad -\frac{GM}{r} \leftrightarrow \frac{Q}{4\pi\epsilon_0 r}$$

2. Differences: gravity only attracts and its potential is always negative; electric forces attract or repel and V can have either sign.
3. Both forces fall as $1/r^2$, so their ratio for two particles does not depend on r . For a proton and an electron:

$$\frac{F_E}{F_G} = \frac{e^2}{4\pi\epsilon_0 G m_p m_e} = \frac{8.99 \times 10^9 \times (1.60 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 9.11 \times 10^{-31}} = 2.3 \times 10^{39}$$

RESULT

same $1/r^2$ form; $F_E \gg F_G$ for particles

03 Key equations

Field strength (definition)

$$E = \frac{F}{q}$$

Coulomb's law

$$F = \frac{Qq}{4\pi\epsilon_0 r^2}$$

Field of a point charge

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

r from the centre

Electric potential

$$V = \frac{Q}{4\pi\epsilon_0 r}$$

zero at infinity; sign of Q

Potential energy of two charges

$$E_p = \frac{Qq}{4\pi\epsilon_0 r}$$

Work done / energy change

$$W = q \Delta V$$

Field and potential gradient

$$E = -\frac{\Delta V}{\Delta r}$$

Uniform field (parallel plates)

$$E = \frac{\Delta V}{\Delta d}$$

The Coulomb constant

$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$$

N m² C⁻²

Symbols and units

Q, q	charges	C	r	distance between centres	m	ϵ_0	permittivity of free space, 8.85×10^{-12}	F m ⁻¹
F	electric force	N	E	electric field strength	N C ⁻¹	V	electric potential	V
E_p	electric potential energy	J	W	work done moving a charge	J	d	plate separation	m
e	elementary charge, 1.60×10^{-19}	C	m_p	proton mass, 1.67×10^{-27}	kg	m_e	electron mass, 9.11×10^{-31}	kg

04 Sample problems with solutions

Sample 1 · Force between two charged spheres

Two small spheres carry charges of +4.0 nC and -6.0 nC, with their centres 5.0 cm apart. Calculate the force between them and state its direction.

Step 1. In SI units (nC = 10⁻⁹ C, 5.0 cm = 0.050 m), Coulomb's law with the size of each charge. Opposite signs, so the force is attractive.

$$F = \frac{8.99 \times 10^9 \times 4.0 \times 10^{-9} \times 6.0 \times 10^{-9}}{0.050^2} = 8.63 \times 10^{-5} \text{ N}$$

Answer: $F = 8.6 \times 10^{-5}$ N, attractive

Unit check: $\frac{\text{Nm}^2\text{C}^{-2} \times \text{C} \times \text{C}}{\text{m}^2} = \text{N} \checkmark$

Sample 2 · The neutral point between two charges

Charges of +2.0 μC and +8.0 μC are 30 cm apart. Find the point between them where the electric field is zero, and calculate the potential there.

Step 1. At distance x from the 2.0 μC charge the two fields are equal and opposite:

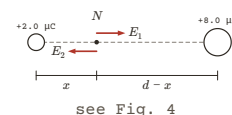
$$\frac{2.0}{x^2} = \frac{8.0}{(0.30 - x)^2} \Rightarrow \frac{0.30 - x}{x} = \sqrt{4} = 2 \Rightarrow x = 0.10 \text{ m}$$

Step 2. Potentials add as scalars:

$$V = 8.99 \times 10^9 \left(\frac{2.0 \times 10^{-6}}{0.10} + \frac{8.0 \times 10^{-6}}{0.20} \right) = 5.39 \times 10^5 \text{ V}$$

Answer: 10 cm from the +2.0 μC charge; $V = 5.4 \times 10^5$ V there

Unit check: $\text{Nm}^2\text{C}^{-2} \times \frac{\text{C}}{\text{m}} = \text{NmC}^{-1} = \text{JC}^{-1} = \text{V} \checkmark$



04 Sample problems with solutions (continued)

Sample 3 · Closest approach of an α -particle

An α -particle (charge $+2e$) with 7.7 MeV of kinetic energy is fired straight at a gold nucleus (charge $+79e$). Calculate how close it gets to the centre of the nucleus.

Step 1. Kinetic energy in joules:

$$E_k = 7.7 \times 10^6 \times 1.60 \times 10^{-19} = 1.23 \times 10^{-12} \text{ J}$$

Step 2. At closest approach it stops for an instant: all the kinetic energy has become electric potential energy.

$$E_k = \frac{Qq}{4\pi\epsilon_0 r}$$

Step 3. r is in the denominator and E_k is the lone term on the other side: swap them in one step (cross-multiplication).

$$r = \frac{Qq}{4\pi\epsilon_0 E_k} = \frac{8.99 \times 10^9 \times (2 \times 1.60 \times 10^{-19})(79 \times 1.60 \times 10^{-19})}{1.23 \times 10^{-12}} = 2.95 \times 10^{-14} \text{ m}$$

Answer: $r = 3.0 \times 10^{-14} \text{ m}$

Unit check: $\frac{\text{Nm}^2\text{C}^{-2} \times \text{C}^2}{\text{J}} = \frac{\text{Nm}^2}{\text{Nm}} = \text{m} \checkmark$

Sample 4 · Two charged spheres hanging from one point

Two identical small spheres, each of mass 0.20 g and charge Q , hang from the same point on insulating threads 0.50 m long. They repel and come to rest with each thread at 15° to the vertical. Calculate Q .

Step 1. Each sphere is in equilibrium. Resolving, $F_T \cos \theta = mg$ and $F_T \sin \theta = F$; dividing the two:

$$F = mg \tan \theta = 0.20 \times 10^{-3} \times 9.81 \times \tan 15^\circ = 5.26 \times 10^{-4} \text{ N}$$

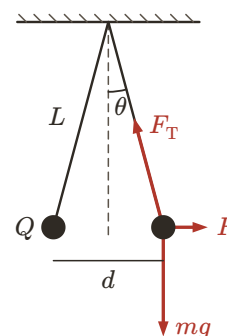
Step 2. Separation of the centres:

$$d = 2L \sin \theta = 2 \times 0.50 \times \sin 15^\circ = 0.259 \text{ m}$$

Step 3. Coulomb's law with equal charges, $F = Q^2 / (4\pi\epsilon_0 d^2)$:

$$Q = d\sqrt{4\pi\epsilon_0 F} = 0.259 \times \sqrt{\frac{5.26 \times 10^{-4}}{8.99 \times 10^9}} = 6.26 \times 10^{-8} \text{ C}$$

Fig. 5 · Two charged spheres in equilibrium



Tension, weight and the electric repulsion balance on each sphere: $F = mg \tan \theta$.

Answer: $Q = 6.3 \times 10^{-8} \text{ C}$ (63 nC)

Unit check: $\text{m} \times \sqrt{\frac{\text{N}}{\text{Nm}^2\text{C}^{-2}}} = \text{m} \times \frac{\text{C}}{\text{m}} = \text{C} \checkmark$

05 Practice problems

- P1** Calculate the force between two protons 1.0×10^{-15} m apart.
Answer: 2.3×10^2 N, repulsive
- P3** Calculate the force on an electron in a field of strength 3.0×10^4 N C⁻¹.
Answer: 4.8×10^{-15} N, opposite to the field
- P5** Calculate the work done moving a $+2.0$ μ C charge from a point at 50 V to a point at 250 V.
Answer: 4.0×10^{-4} J
- P7** An isolated sphere of radius 0.15 m is charged to a potential of 3.0×10^5 V. Calculate its charge.
Answer: 5.0×10^{-6} C
- P9** Calculate the ratio of the electric force to the gravitational force between two protons.
Answer: 1.2×10^{36}
- P2** Calculate the electric field strength 0.20 m from a $+5.0$ μ C point charge.
Answer: 1.1×10^6 N C⁻¹, away from the charge
- P4** Calculate the potential 0.10 m from a -3.0 nC point charge.
Answer: -2.7×10^2 V
- P6** The distance from a point charge is doubled. By what factor do F, E and V change?
Answer: F and E to $\frac{1}{4}$; V to $\frac{1}{2}$
- P8** Along a field line the potential falls by 60 V over 2.0 mm. Calculate the field strength there.
Answer: 3.0×10^4 V m⁻¹
- P10** Calculate the electric potential energy of an electron and a proton 5.3×10^{-11} m apart.
Answer: -4.3×10^{-18} J

06 Other useful information

Common mistakes

- ✗ **Wrong:** Add the fields of two charges as plain numbers.
- ✓ **Right:** E is a vector: draw each field's direction first. V is a scalar: add with signs.
- ✗ **Wrong:** Use the distance from the surface of a charged sphere.
- ✓ **Right:** r is measured from the centre.
- ✗ **Wrong:** Electric potential is always negative, like gravitational potential.
- ✓ **Right:** V is positive near a positive charge, negative near a negative one.
- ✗ **Wrong:** Where E = 0, V must be 0 too.
- ✓ **Right:** No: at the neutral point between like charges E = 0 but V is not.
- ✗ **Wrong:** Forget to convert nC and μ C.
- ✓ **Right:** nC = 10^{-9} C, μ C = 10^{-6} C. Square r in metres.

Exam tips

- Sketch the charges and draw each field arrow (away from +, towards -) before calculating.
- Neutral point between like charges: $Q_1/x^2 = Q_2/(d-x)^2$, then take square roots.
- Closest approach and similar energy questions: kinetic energy lost = potential energy gained.
- Hanging charged spheres: resolve forces, $F = mg \tan \theta$, then Coulomb's law.

Explore

- [Coulomb's Law Lab \(simulation\)](#)
- [Practice: Coulomb's law \(98 questions\)](#)
- [Practice: field strength of a point charge](#)
- [Gravitational fields cheat sheet](#)

What the syllabus asks

- 9702 topic 18, Electric fields: $E = F/q$, field lines, Coulomb's law, $E = Q/4\pi\epsilon_0 r^2$, $V = Q/4\pi\epsilon_0 r$, E as the negative potential gradient, $E_p = Qq/4\pi\epsilon_0 r$. Coursebook Chapter 22.
- A Level only: not part of AS.
- For a point outside a charged conducting sphere, treat its charge as a point charge at its centre.
- The matching gravity results are in Chapter 17.